Effects of perturbing forces on the orbital stability of planetary systems

L.Kiseleva-Eggleton[1]

IGPP, Lawrence Livermore National Laboratory, L-413, 7000 East Ave, Livermore, CA 94550

lkisseleva@igpp.uclinl.org

and

E.Bois

Observatoire de Bordeaux, 2 rue d'Observatoire, B.P.89, F-33270, Floirac, France

bois@observ.u-bordeaux.fr

Received ______________________; accepted ______________________

[1]Also Dept. of Physics, University of California, Davis

ABSTRACT

We consider dynamical effects of additional perturbative forces due to the non-point mass nature of stars and planets: effects such as quadrupolar distortion and tidal friction in the systems of exo-planets. It is shown that these forces should not be neglected while modelling the dynamics of planetary systems, especially taking into account the undefined real masses of the planets due to unknown orbital inclinations and the unsatisfactory application of Keplerian fits to the radial velocity data in multiple planetary systems.

Subject headings: celestial mechanics, stellar dynamics - planetary systems - stars:individual (υ Andromedae)

1. Introduction

About 50 extrasolar planets have been discovered so far, among them: at least three confirmed planetary systems around the stars υ And with three Jupiter mass planets (e.g. Laughlin & Adams 1999, Rivera & Lissauer 2000); HD 83443 with two Saturn type companions (Mayor et al. 2000); and GJ 876 orbited by two resonant planets (Marcy et al. 2001). In addition there are two substellar-mass companions orbiting HD 168443 (Marcy et al. 2001, Udry et al. 2001), a system of Earth-mass planets around pulsar PSR 1257+12 (e.g. Konacki et al. 1998), as well as yet-to-be-confirmed systems such as a second planet around 55 Cnc (Jayawardhana et al. 2000), planets orbiting Lalande 21185 (Walker 1996), and three Earth-type planets around pulsar PSR 1828-11. No doubt the number of detected planetary systems will continue to increase rapidly due to the improvement of observational detection techniques and to new space missions (e.g. Kepler, COROT, FAME, SIM etc.) scheduled for the next few years.

The majority of members of new planetary systems have eccentric orbits. This may increase the dynamical interaction between components and make systems chaotic and potentially hierarchically unstable, ending up with an ejection of one or more (in systems with more than 2 planets component) to infinity. Other main factors which add to uncertainty of the dynamical state of the systems are **(a)** the unknown value of orbital inclination i, which allows us to know only the lower limit of planetary masses m_o from the function $m_o = m_p \sin i$, and leaves the real mass m_p as almost a free parameter (for attempts of i estimation see Gatewood et al. 2001, Pourbaix 2001), and **(b)** the unknown relative inclination between planetary orbital planes. Also, usually the orbits in planetary systems are not Keplerian because of mutual interaction between planets. Therefore standard Keplerian fits to velocity observations are strictly speaking not valid. This point was addressed by Laughlin & Chambers (2001) who suggested a new four-stage procedure for the determination of dynamical parameters of multiple planetary systems. This procedure includes multiple-Keplerian fits using a semi-analytic scheme followed by a final self-consistent polish with N-body interations. This method should be a substantial improvement to existing fitting techniques. However, in cases when close approaches of two or more components are possible, one should take into account the non-point-mass nature of the bodies and, as a consequence, the following perturbations to Newtonian gravity: (**a**) the quadrupolar distortion (QD) of the bodies due to their mutual gravity; (**b**) the further quadrupolar distortion due intrinsic spin of the components; (**c**) tidal friction (TF); (**d**) General Relativity. In this paper we present a few examples showing the influence of two of these perturbations - (**a**) and (**c**)- on dynamical stability of systems with two planets.

2. The model

In order to estimate the dynamical effects of quadrupolar distortion of interacting bodies due to their mutual distortion (QD) and to tidal friction (TF), based here on the near-equlibrium approximation, on the planetary systems treated as N-body systems with extra forces due to QD and TF in addition to the Newtonian gravity, we applied the following formulation for the force $\mathbf{F}_{ij}$ of one body on the other, developed by Kiseleva, Eggleton & Mikkola (1998) - hereinafter KEM98:

$$\begin{aligned}\mathbf{F}_{ij} = &-\left[\frac{Gm_im_j}{r_{ij}^3} + \frac{6G(m_j^2A_i + m_i^2A_j)}{r_{ij}^8}\right]\mathbf{r}_{ij} \\ &+\left[\frac{27}{2}\frac{\sigma_i m_j^2 A_i^2 + \sigma_j m_i^2 A_j^2}{r_{ij}^{10}}\,\mathbf{r}_{ij}.\dot{\mathbf{r}}_{ij}\right]\mathbf{r}_{ij} \qquad ,\end{aligned} \tag{1}$$

where

$$A_k = \frac{R_k^5 Q_k}{1-Q_k}, \qquad \sigma_k = \frac{\alpha_k}{Q_k^2 m_k R_k^2}\sqrt{\frac{Gm_k}{R_k^3}} \qquad ,$$
$$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j. \tag{2}$$

Here R_k are the stellar or planetary radii, and α_k are the dimensionless dissipation rates for the two bodies. Q is a version of the apsidal motion constant, a dimensionless measure of the distortability of the body (star or planet). KEM found that for a polytrope of index n in the range $0 \leq n \leq 4.95$, Q can be approximated by the interpolation formula

$$Q \;\approx\; \frac{3}{5}\left(1-\frac{n}{5}\right)^{2.215} e^{0.0245n-0.096n^2-0.0084n^3} \tag{3}$$

We applied this model to the υ And planetary system. Recently Jiang & Ip (2001) confirmed once more that the innermost planet does not affect very much the dynamics of the middle and outer planets, and so we ignored it in our simulations. For calculation we used the regularized CHAIN method (Mikkola & Aarseth 1993) with perturbations. The

actual numerical integration of the equations of motion are carried out by a Bulirsh-Stoer integrator with a timestep accuracy of 10^{-14}.

In our simulations we always used initial orbital parameters for orbital periods and eccentricities from the Lick Data (Butler et al. 1999, Rivera & Lissauer 2000): $P_c = 242$ days, $e_c = 0.23$ for the middle planet **C** and $P_d = 1269$ days, $e_d = 0.36$ for the outermost planet **D**. We always started simulations with all three components positioned in the same plane at apoastrons of their orbits and with the two orbits out of phase with each other by 90 deg (so we did not use the observed values of ω). Orbital parameters for the υ And planetary system are not very precisely defined and may vary rather significantly (particularly the eccentricities) during the lifetime of the system (see below). Note, however, that differences between values of orbital parameters given by different authors may not be explained by real changes during a short observational time of a few years. Any real and significant changes require a time scale of hundreds years.

3. Results

In our first set of simulations we use the nominal values of the planetary masses from the Lick Data: for $\sin i = 1$ $m_c = 1.98 M_J$, $m_d = 4.11 M_J$. The adopted mass of the star was $M = 1.3\, M_\odot$. The relative inclination between the two orbital planes was taken to be $\phi = 1$ deg (models with higher relative inclinations were also tested but we will discuss those results in our next paper). For this set of parameters the system appear to be hierarchically stable over at least 10^7 years, despite both orbital eccentricities fluctuating almost quasi-periodically within significant ranges: $e_c^{max} - e_c^{min} \approx 0.5$, $e_d^{max} - e_d^{min} \approx 0.15$. These results are in good qualitive agreement with other simulations (Laughlin & Adams 1999, Rivera & Lissauer 2000). Neither TF or QD or their combination changes notably (except very minor details) the orbital evolution of this system.

However, because of the unknown values of the inclination of the orbital planes of the planets i_c and i_d, which can differ very much from each other, even the mass hierarchy of the υ And planetary system is questionable. If one accepts the values of $i_c = 173.7\,\mathrm{deg} \pm 3.8\,\mathrm{deg}$, $i_d = 28.7\,\mathrm{deg} \pm 16.8\,\mathrm{deg}$ proposed by Pourbaix (2001) with a big uncertainty, this will lead to a very different system with component **C** being no longer a planet ($m_c \approx 18 M_J$), and $m_d \approx 8.5 M_J$ (so the mass hierarchy is now changed). In agreement with earlier stability analyses (Stepinski et al. 2000, Rivera & Lissauer 2000) the new system is highly unstable and disintegrates ejecting the component **D** within the first 1000 years. The addition of QD increases significantly the lifetime of the system, although it does not change the chaotic and unstable character of its dynamics.

In order to study more systematically the effects of QD and TF (in this work we study these effect separately) on dynamical stability of planetary systems unstable in point mass approximation, we exchanged the masses of planets **C** and **D** in the υ And system, leaving their intial eccentricities and orbital periods unchanged. Without QD and TF this system is unstable ejecting the less massive ($m_d = 1.98 M_J$) component **D** within $\sim 3 \times 10^5$ years (top panel of Fig. 1). QD and TF may significantly change the dynamical evolution of the system. The scale of these changes depends very much on the chosen values of coefficients Q_i and α_i from Eq 2. The value of Q_i is defined by the polytropic index n_i of the body (Eq 3). For the presumably radiative star of $1.3\,M_\odot$ we assume $n \sim 3$ with corresponding

Fig. 1.— Eccentricity of the outer planet in the υ And planetary system with masses of planets **C** and **D** exchanged. The system is unstable without QD and TF (top panel) and with relatively weak QD and TF (second and fourth panels), although QD increases the life time of the system. QD with coefficients $Q_* = 0.08$ and $Q_c = Q_d = 0.2$ and TF with $\alpha = 10^{-4}$ make the system stable over the considered time interval (10^6 years), with quasi-periodic fluctuations of e_d (third and bottom panels)

$Q_* \approx 0.02$, and for planets we took $Q \approx 0.2$ ($n \sim 1.5$). The dynamics of the system during the first $\sim 1.5 \times 10^5$ years is significantly less chaotic than without QD and the planet D is not ejected to infinity until $t \sim 5 \times 10^5$ years. But the situation changes really dramatically when we increase the value of Q_* to 0.08 (so assuming a more convective interior of the star). The third panel of Fig. 1 show that in this case the system displays hierarchically stable dynamical behaviour over at least 10^6 years, with quasi-periodic fluctuations of both orbital eccentricities similar to ones of original υ And system.

The dynamical effect of tidal friction depends on its coefficients α_i. Kiseleva et al. (1998) found that for stars similar to the ones in the λ Tau triple system (Fekel & Tomkin 1982) the most likely $\alpha \sim 10^{-5}$. However, for planets α can be significantly larger. We tested our model with $\alpha = 10^{-5}$ and $\alpha = 10^{-4}$ for all 3 bodies. The results shown on the two lower panels of Fig. 1 are totally different. $\alpha = 10^{-5}$ does not seem to improve the stability of the system. However, for $\alpha = 10^{-4}$ the bottom panel presents once more a hierarchically stable system with quasi-periodic behaviour of its orbital parameters such as e_d. Dynamical evolution of models with strong QD ($Q_* = 0.08$) and with strong TF ($\alpha = 10^{-4}$) over 10^6 years look in this case remarkably similar, despite different dynamical properties of these perturbations: TF is a dissipative force with respect to the total orbital energy and QD is conservative. However, such a similarity does not appear in other cases (see below), and we suspect that over longer time the evolutionary patterns with QD and with TF will diverge.

We also studed models of υ And with $\sin i = 0.33$ for both external planets **C** and **D**, so the mass hierarchy of the nominal system is preserved and $M_c = 6.53 M_J$, $M_d = 13.56 M_J$. The results are shown on Fig.2. TF and especially QD significantly increase the lifetime (upto correspondingly $\sim 5 \times 10^4$ and $\sim 1 \times 10^5$ yrs) of this unstable system.

4. Conclusions and prospects

Our strongest conclusion is that the effects of perturbative forces such as quadrupolar distortion and tidal friction should not be neglected when investigating numerically the dynamical properties of extra-Solar planetary systems, especially when their hierarchical stability is questionable and even weak additional forces may change the qualitive character of their dynamics. In our examples QD always had a stabilizing effect, but we can not claim that it always works this way and more studies are needed.

We did not consider here the contribution of General Relativity and intrinsic rotation of the bodies which under some circumstances may be important. In any case, because of problems with the reliable determination of orbital parameters which cannot be consider as Keplerian in N-body systems, it would be very useful to apply a good dynamical chaos indicator (Lyapunov-type exponent, for example), which can distinguish a long-term dynamical instability from relatively short-term integrations, given a good-size sample of possible initial parameters. One such indicator was suggested recently by Cincotta & Simó (2000). Our first attempts to apply it to extra-Solar planetary systems are very encouraging and we are going to discuss the results in our next paper (Goździewski, Bois, Maciejewski & Kiseleva-Eggleton, in preparation).

The authors are grateful to the John Templeton Fondation for the grant which supported the publication of this paper in ApJ. LK-E thanks the Bordeaux Observatory for

Fig. 2.— Eccentricities of planets C (left) and D (right) in υ And with $\sin i_c = \sin i_d = 0.33$. Without TF and QD the system is destroyed within the first 20000 years (top panel). TF (middle panel) and especilly QD (bottom panel) significantly increase the lifetime of the system. The system with QD actually disintegrates after $\sim 10^5$ yrs.

hospitality.

REFERENCES

Butler, R.P., Marcy, G.W., Fischer, D. A., Brown, T. M., Contos, A. R., Korzennik, S. G., Nisenson, P. & Noyes, R. W., 1999, ApJ, 526, 916

Cincotta, P. M. & Simó, C., 2000, A&ASS, 147, 205

Gatewood, G., Han, I. & Black, D., 2001, ApJ, 548, L61

Fekel, F. C. & Tomkin, J., 1982, ApJ, 263, 289

Jayawardhana R., Holland W., Greaves J., Dent W., Marcy G., Hartmann L. & Fasio G., 2000, ApJ., 536, 425

Jiang, I.-G. & Ip, W.-H., 2001, ApJ, submitted

Kiseleva, L. G., Eggleton, P. P. & Mikkola, S., 1998, MNRAS, 300, 292

Konacki M., Maciejewcki A. & Wolszczan A. 1998, ApJ. 513, 471

Laughlin, G. & Adams, F. C., 1999, ApJ, 526, 881

Laughlin, G. & Chambers, J. E., 2001, ApJ, in press

Marcy, G.W., Butler, R.P., Fisher, D., Vogt, S., Lissauer, J. J. & Rivera, E., 2001, ApJ, submitted

Marcy, G.W., Butler, R.P., Vogt, S., Liu, M. C., Laughlin, G., Apps, K., Graham, J. R., Lloyd, J., Luhman, K. L. & Jayawardhana, R., 2001, ApJ, submitted

Mayor, M., Naef, D., Pepe, F., Queloz, D., Santos, N. C., Udry, S. & Burnet, M., 2000, in Planetary Systems in the Universe, Eds. A. Penny, P. Artymowicz, A.-M. Lagrange and S. Russel, ASP Conf. Ser

Mikkola, S. & Aarseth, S., 1993, Celest.Mech., 57, 439

Rivera, E.J. & Lissauer, J. J., 2000, ApJ, 530, 454

Pourbaix, D., 2001, A&A, submitted

Stepinski, T. F., Malhotra, R. & Black, D. C., 2000, ApJ, 545,1044

Walker, G., 1996, Nature, 382, 23